

Data Analysis in Archery

A guideline by Archery Analytics GmbH, © 2020-2023

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1. Introduction

The autonomous RyngDyng[®] spotters of Archery Analytics record all arrow positions in a very precise and accurate way. They further can connect the measured positions with the settings and information provided by the archers. All data are stored and hence available for later analysis. This unique combination provides entirely new possibilities of which any archer can directly benefit – be it to improve the shooting technique or to find the ideal configuration of the material.

Intention of this document is to introduce a toolbox explaining the various ways of analysis and demonstrating how you as an ambitious archer can make use of it during training, preparation for competition and in the competition itself.

2. Why to Use Statistics?

The assessment of many recorded arrows is based on statistics and data analysis.

All archers have a basic understanding of this fact as they are aware it is not sufficient to just shoot three arrows and then depending on the result you are going to adapt the sight. Further, by intuition, archers know that the term „grouping“ gives an indication about the position and the formation of several arrows. The key figure *points per arrow* is relevant information for reporting the results of a competition and also constitutes a statistical indicator.

However this is just the beginning. In fact, the shooting result on the target face needs to be considered consequently as the result of a statistical process. You as the archer may be able to influence the variables of the statistical process by practicing the shooting technique or by tuning the material. Nevertheless, it is and remains a statistical process. Even with the best archers the arrow positions on the target face vary in a statistical manner and one is just unable to repeat a constant shot with the very same arrow position again.

The difference, however, between good and very good archers is that the latter ones have less variations in the parameters of the shooting process and therefore keep the variations from shot to shot, end to end, day to day at a very minimum. These very experienced archers intuitively know what the potential causes might be when the results tend to get worse and which means they should take for correction.

The aim of this guideline therefore is to illustrate such correlations and contexts and make them comprehensibly to many archers.

By means of the Archery Analytics evaluation software we try to support archers in an optimal way. The software is integrated into our RyngDyng App[®] and conducts the data analysis in your personal account on www.archery-analytics.com.

3. Statistical Basics

a. Number of Arrows

The simplest statistical metric is the **Number of Arrows**, which are shot. During competitions the number is given and structured into **ends** and rounds. During training, the number of arrows can be determined freely, the division into ends however remains as you need to pull the arrows again.

Any book about the basics of archery training recommends to note down the number of arrows shot for all training units in your personal training records.

But did you ever wonder why you should do that? A common explanation is that preparation for a competition shall include a certain amount of training arrows and via the recording you can check whether the recommendation is being followed.

Wouldn't it be more interesting, if you could analyze whether by increasing the number of training arrows your shooting result improves - in the sense of a smaller variation of the arrow positions - (see chapter f The Arrow Grouping Indicator) or does a too high amount of training arrows before the competition lead to the opposite effect, i.e. the shooting result does not get any better or even worse?

Exactly this is one of the possible findings from analyzing the data recorded by the autonomous spotter, RyngDyng®.

When checking the number of arrows from week to week and relate them with your respective Arrow Grouping Indicator AGI, you can determine what your personal optimal amount of training arrows is when preparing for competition. It is also possible to check whether the time profile of the training intensity leads to an optimal result on competition day or whether the optimum has already been reached two days before and the degression has already begun.

b. Number of points, points per arrow

A metric which is collected in every competition is the **Total Number of Points**. This is the most relevant indicator when it comes to scoring and rankings.

Besides number of points, World Archery also shows **Points per Arrow**. This key figure allows archers to be compared independently of the number of competitions participated. As a precondition, only archers within the same competition class shall be compared. The reason is that this figure assumes different values depending on the kind of competition. Compound and recurve archery differentiate substantially in this respect.

Points per Arrow is only suited partially for material tuning or when practicing the shooting technique. It rather has a benefit when specifically simulating competition situations.

So when recording data with RyngDyng®, you can indicate that the following ends are part of a competition situation. Later on, you will be able to compare the computed figure **Points per arrow** with the actually achieved value during the actual competition. In order to assess and monitor the progress of your capability during competitive situations, a long-term observation of this metric makes a lot of sense.

c. Group Center, Shift of the Arrow Group

Every archer is well aware of the following situation: Before pulling the arrows of an end you look at the arrow group and notice that the position of the center of the arrow group doesn't correspond with the center of the target face. You are wondering about the reasons why and if it wouldn't be better to adjust the sight.

So now, what exactly is the **Group Center**? As long as the arrow positions cover a reasonable uniform circular disc, you would call the center of that circle the group center. However, most of the time another situation prevails: Where is the center when some arrows are tightly positioned next to each other and a few deviate to the top or to the left?

The answer is again statistics: The coordinates of the group center are the arithmetic means of the coordinates of the arrow positions.

The following image exemplifies.

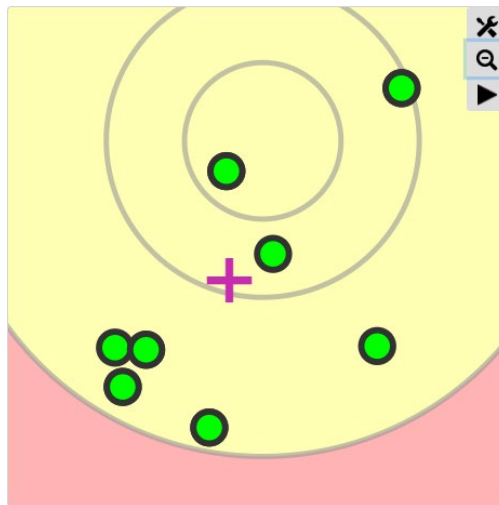


Figure 1: Group Center

The image depicts eight arrows (marked by the green dots) inside the gold of a 60 cm target face. The cross marks the statistical center of this group.

The Archery Analytics evaluation software computes the group center for any amount of arrows and marks the center with a cross. This is the well-defined group center independently of the patterns formed by the arrows.

In this way you can clearly identify the **Shift of the Group**, i.e. the deviation of the arrow group from the center of the target face. This deviation in the evaluation is specified as horizontal and vertical shift in cm. In the above mentioned example the values are -0.6 cm horizontally (i.e. to the left) and -2.7 cm vertically (i.e. down). Positive values correspond to a shift to the right respectively up.

d. Horizontal and Vertical Dispersion

The following three images show arrow groups consisting of 12 arrows each. The group centers (crosses) are approximate at the same position. But where is the difference?

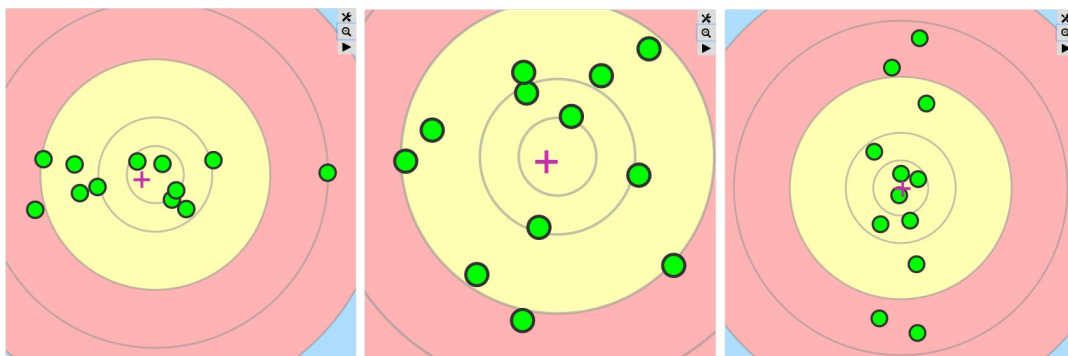


Figure 2: Same Group Center – Varying Dispersions

The difference lies in the so-called **Dispersion**. The Archery Analytics evaluation software computes the respective horizontal as well as the vertical dispersion for every group of arrows.

The computed dispersion of the three depicted arrow groups are:

	Left	Center	Right
Horizontal Dispersion	5.7 cm	4.3 cm	1.3 cm
Vertical Dispersion	1.3 cm	4.6 cm	6.5 cm

When examining the values for dispersion, you can clearly see that in the left image a strong horizontal dispersion is at hand, whereas the vertical one is rather small. The opposite effect can be observed in the right image. The middle one illustrates a horizontal and vertical dispersion which are about the same. Based on the fact that horizontal and vertical dispersion deviate from each other, clear conclusions regarding potential imperfections can be drawn. This is explained further in chapter 5.

The statistical metric as the basis for dispersion is the so-called *standard deviation*. Besides the dispersion in cm, our Archery Analytics software computes the *normalized dispersion*. This metric is generated by dividing the dispersion by the distance at which you shoot. Like that, a metric is created constituting a measure for angle dispersion and not depending on the distance. This metric is of benefit when doing an analysis of imperfections, which can be found e.g. chapter 5.b *Release Faults*.

e. Scatter Ellipse

Both images that follow demonstrate two arrow groups with 12 arrows each. The groups have about the same group center, but also the same horizontal and vertical dispersion:

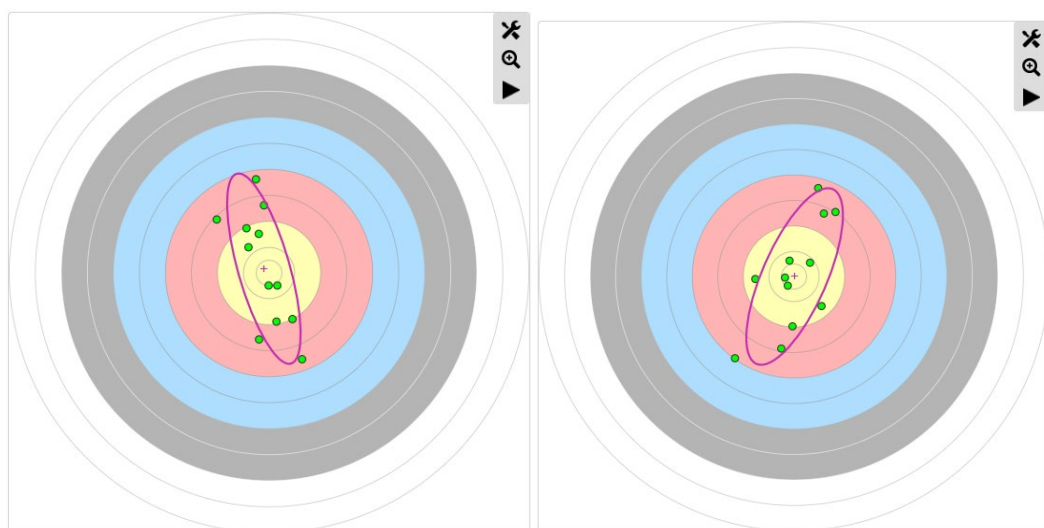


Figure 3: Rotated Scatter Ellipses

Yet they differ significantly. The difference is evident in the drawn **Scatter Ellipse**.

As you can see in the image on the left side, the scatter ellipse is inclined in another way as the ellipse on the right image. So there must be different causes why in the left image the arrows tend to the left the higher they stick in the target face, whereas in the right image the effect is reversed. In chapter 5.c you will get to know that this might be caused by an uneven finger pull at the bow string.

The Archery Analytics evaluation software computes for every displayed arrow group comprised of at least three arrows the respective scatter ellipse. Its center point is identical with the group center and is indicated by the cross.

The scatter ellipse is not only helpful when it comes to the analysis of certain shooting faults, but it also shows at a glance whether the horizontal and vertical dispersion is equal or not. In case of an equal horizontal and vertical dispersion the ellipse becomes a circle – as can be seen in the subsequent example of an overlay of three ends:

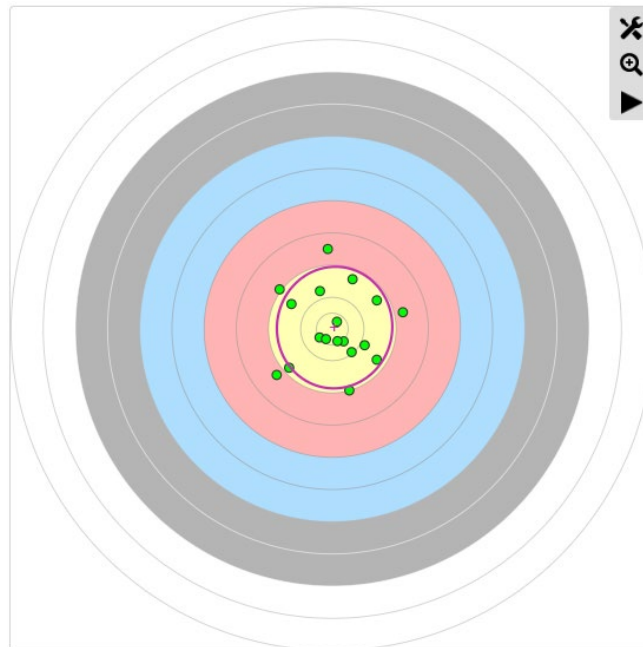


Figure 4: Scatter ellipse as a circle

In this example the horizontal dispersion accounts to 4.4 cm, the vertical one to 4.6 cm.

The size of the ellipse is calculated in a way that about 90% of the arrows lie within the ellipse and 10% outside. It results from a statistical computation and is therefore not identical with a line encompassing all arrows (the so-called *convex hull*, which corresponds to a band wrapped around the arrow group).

It therefore gives evidence about the majority of the arrows; outliers have only a proportionate effect on the ellipse, which is revealed by the next image:

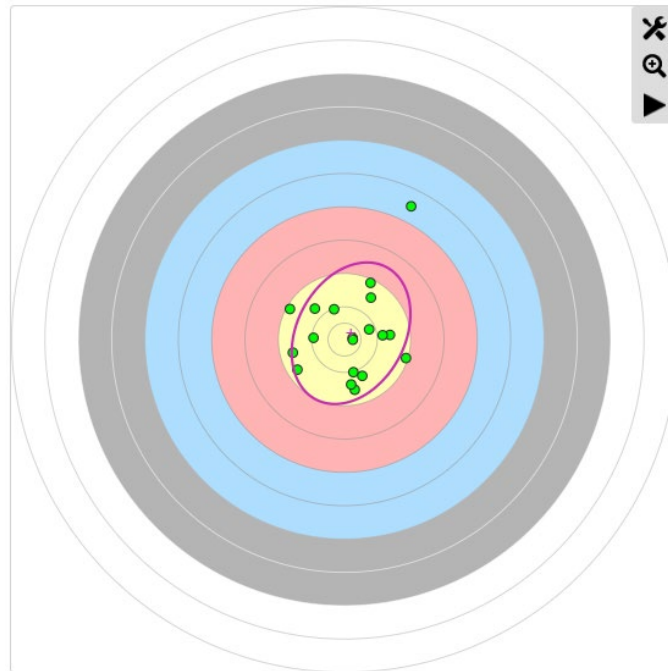


Figure 5: Effect of outliers

While the outlier in ring six has an obvious effect on the scatter ellipse, it needs to be stated that it is only one arrow out of 18, which diminishes its influence from a statistical point of view. However, if there would be only five arrows in the gold, the effect of the outlier on the dispersion and the corresponding scatter ellipse would be respectively bigger.

f. Arrow Grouping Indicator (AGI)

The most important number while being in training and for tuning of the bow is the **Arrow Grouping Indicator (AGI)**, specifically developed by Archery Analytics. Intention is to give evidence about the grouping of arrows. The AGI has the following characteristics:

- It is a number between 0 and 100.
- 100 implies: all arrows have the exact same position.
- 0 implies: the arrows are scattered at maximum on the target face (referring to the Figure face 122 on 70 m distance).
- The AGI can be computed for any arrow group consisting of at least three arrows.
- The AGI is irrespective of the shooting distance. It constitutes a parameter for the quality of the shooting result.
- The AGI is irrespective of the position of the group center. It just depends on how the arrows are positioned to each other.
- The AGI is irrespective of a rotation of the arrow group around the group center. (see *Figure 3: Rotated Scatter Ellipse*).

The fact that the AGI is irrespective of the distance is illustrated by the following draft:

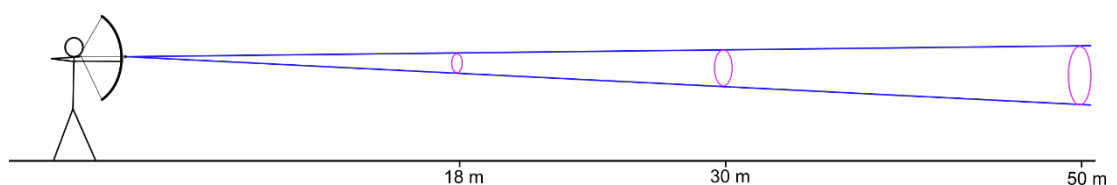


Figure 6: AGI and distance

Imagine, in case that all arrows fly within the corridor marked in blue color, they would then hit within the purple scatter ellipses at the various distances. Consequently, for the grouping of the arrows it follows that a smaller scatter ellipse on a short distance shows the same inaccuracy as a bigger scatter ellipse on a larger distance.

The AGI is based on the size of the scatter ellipse. The ellipse however needs to be converted to a uniform distance proportionally. Thus, the AGI becomes irrespective of the distance and represents ultimately a measure for the statistical angle dispersion.

Due to its specific properties, the AGI becomes an utterly useful instrument when measuring the quality of the shooting result and for the tuning of the material. The other chapters of this guideline will handle this topic.

In order to not betray too much at this point: in case you wish to get all your arrows into the gold, then your AGI should be reliably amount to at least 95, and in addition forming a ring like scatter ellipse. Furthermore, you should be capable of placing the group center in the center of the target face. Out of statistics and experience we know that the latter one is the easier part compared to achieving an AGI of 95.

g. Distribution of Hits

In addition to the center point of the arrow group and the dispersions (shown as a scatter ellipse), you can also analyze the distribution of the hits. The so-called [boxplots](#) are used for this purpose, and they have a straightforward interpretation. The following example shows a lying (horizontal) boxplot below the hits and a standing (vertical) boxplot to the left of them:

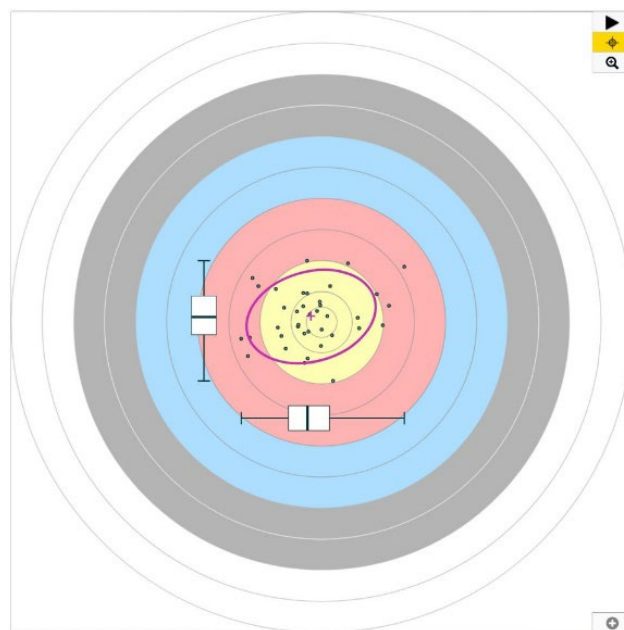


Figure 7: Boxplots

The interpretation is as follows: Each rectangle (= box) marks the area within which half (50%) of the hit positions are located. The ends of the 'whiskers' mark the extreme vertical and horizontal positions of the arrows.

The thick line inside the box is called the median and divides the hit positions in half (left - right and up - down). Extending the vertical and horizontal median gives this picture:

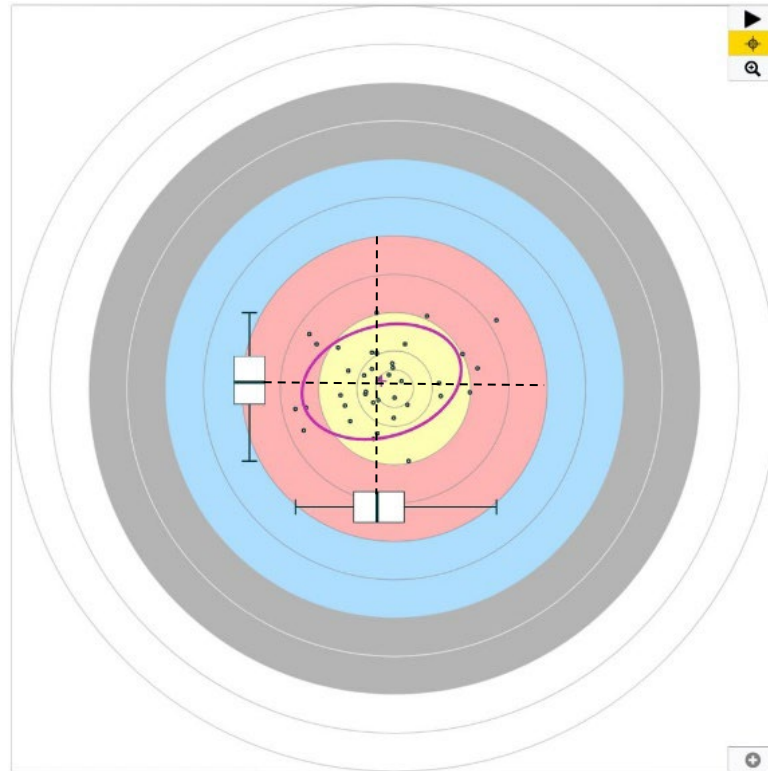


Figure 8: Median

Left and right of the vertical line are 50% of the hits each. Likewise above and below the horizontal line. Incidentally, the intersection of the median lines does not have to be identical to the center of the group. These only coincide if the distribution of hits is even, as shown in Figure 8.

If the median is not approximately in the middle of the respective box, the hits are unevenly (asymmetrically) distributed. From this fact, you can draw important conclusions about the accuracy and faults of the shooting process.

Here is an example of an asymmetric hit distribution:

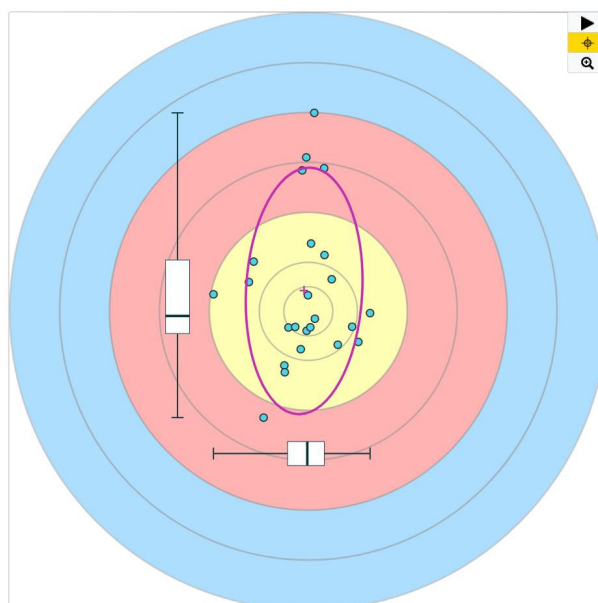


Figure 9: Vertically asymmetric hit distribution

The vertical boxplot indicates that the top 50% of hits extend farther up than the bottom 50% down. The vertical hit pattern is therefore asymmetrical while the horizontal one is symmetrical. From the elongated scatter ellipse, we can see that there is increased vertical dispersion. However, only the vertical boxplot shows that this dispersion is not uniform, but that the hits in the lower half are denser than in the upper half.

The cause of this hit pattern is most likely an uneven pressure point in the bow hand. The pressure point moves down repeatedly, so that the hits land higher. This does not happen all the time though, so there are fewer hits at the top than within the target center.

h. Significance and Trust

As we can have several ends overlaid in our analysis software, it enables you as the user to view many arrows from different ends as one arrow group. This is an asset as it increases the trust in the computed metrics tremendously.

Let's have a look at the example of the shift of the arrow group. You should only consider an adjustment of the sight once you have recorded sufficient arrows representing a meaningful group, and with this large number of arrows a distinct shift of the group center has become apparent. You know this because a shift of e.g. only six arrows can happen by mere chance, whereas with the subsequent six arrows the outcome might be different again.

As an example of the overlay of several ends, please see the following image with 36 arrows from six ends each with six arrows respectively. The group center (cross) here doesn't reveal any evidence for an explicit group shift – therefore, no need to act.

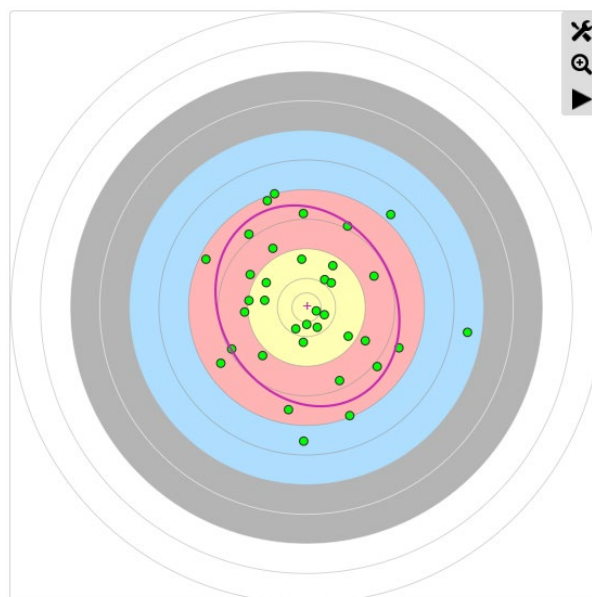


Figure 10: Group Center with six Overlaying Ends

The trust in a statistical evidence is called *Significance*. The significance of a computed metrics is higher the more arrows are depicted and analyzed at the same time.

We achieve a very high significance because our Archery Analytics spotter, RyngDyng© records many ends, stores all data for further analysis and enables the data to become overlaid as one image. If you are checking your results only with Figure to one end (e.g. when pulling the arrows), such a high significance (trust) cannot be achieved:

No recorded arrow positions – no significance.

4. Tuning of the Material Settings with RyngDyng[©]

a. Basic Settings

Before beginning with the tuning of particular parameters, some basic settings need to be carried out first. It only makes sense to optimize on individual parameters when the basic settings have been already well adjusted.

These basic settings for recurve bows include

- Centering of the limbs
- Brace height
- Tiller and tiller difference
- Height of arrow rest
- Nocking point height
- Centering of the resting arrow by the button
- Centering of the sight
- Hardness of the button spring

We strongly recommend to implement these settings in the same sequence as described above.

When centering the limbs, bear in mind that the string takes course right in the middle of the limbs and the axis of the riser. This is done by means of a centering gauge on the limbs and by optical control of the string position.

Regarding the brace height you can orientate at the range provided by the manufacturer of the bow. Some common recommendations are:

Size of the Bow	Range for Brace height
64"	210 – 216 mm
66"	213 – 219 mm
68"	216 – 222 mm
70"	217 – 225 mm

For the basic setting it is best to just take a value in the middle of the recommended range.

The tillers of the limbs are advised to be adjusted in a way that the desired draw weight is reached when in full draw and with a tiller difference of about 4-6 mm.

The height of the arrow rest is to be adjusted so that the center of the button pin is aligned with the middle of the arrow shaft.

The nocking point height is also set to a suggested value of between 8 and 12 mm.

The tip of the resting and nocked arrow is centered with the help of the button pin or is slightly shifted outwards (max. 1-2 mm at the tip).

The sight is getting centered, i.e. the sight notch is positioned in the central vertical layer consisting of string and central axis of the bow riser.

The spring hardness of the button should be within the „working range“ which can be detected by shooting some arrows on 18 m distance: Initially, set the spring hardness to very strong by turning the handle of the button inwards. With this setting and the centered sight and using a right hand bow, the arrows should hit the target left of the targeting point (with

a left hand bow right of the targeting point). Thereafter, soften the button step by step by half a turn until the arrows hit approximately the vertical line through the targeting point (please note it is not yet relevant whether it is underneath or above).

b. Brace Height

After you have adjusted the basic settings of the bow, you can continue by improving the settings of the brace height. Usually, once the ideal brace height has been found, it is hardly ever getting altered again – even if you wish to change tiller, nocking point height or the button strength afterwards.

The brace height is decisive when it comes to determining the exact point in time when the arrow leaves the string. In case the timing is not right, clearance problems can occur, i.e. the arrow touches parts of the riser leading to an improper trajectory. Furthermore, the initial speed of the arrow depends on the brace height as the duration of the accelerating power is dependent on the brace height.

Now, the intriguing point is that the optimal brace height can be found by means of grouping tests. The *Arrow Grouping Indicator* AGI (see chapter 3.f) expresses the quality of a group with only one specific number – independent of the shooting distance - so that the AGI represents the best suited metric to conduct these tests.

Besides the grouping tests, the sound of the bow when shooting is an often used indicator. However, the assessment of its sound presumes some experience as it may easily mislead. As a rule of thumb it applies that if the arrow shot doesn't sound right, the brace height might be wrongly set.

On the contrary if the shot sounds right, the brace height is not necessarily optimal. Because it is only optimal when the AGI value is optimal, too.

In order to find the optimal brace height, the range for this parameter needs to be browsed. Depending on how accurately you wish to do it, the brace height will get adjusted by intervals of 1 or 2 mm, beginning with the smallest value ending at the highest, and always aiming at the very same point on the target. Whether the arrows hit that point or not is fully irrelevant. The grouping of the arrows matters.

We recommend to conduct this test on a distance of 30 m - if possible. This distance is well suited because other effects, which can cause an improper arrow flight, can act evenly.

The RyngDyng® App supports the mentioned browsing of the parameter range insofar as for every brace height an arbitrary number of arrows is recorded and the result is presented then in a clearly arranged chart.

The following image shows a test, where four different brace heights were selected with each 18 arrows shot. As we have selected the *Grouping test* in the RyngDyng© App, the below chart is depicted with the AGI values at the respective brace heights. It can be clearly seen: The best AGI value is at a brace height of 224 mm.

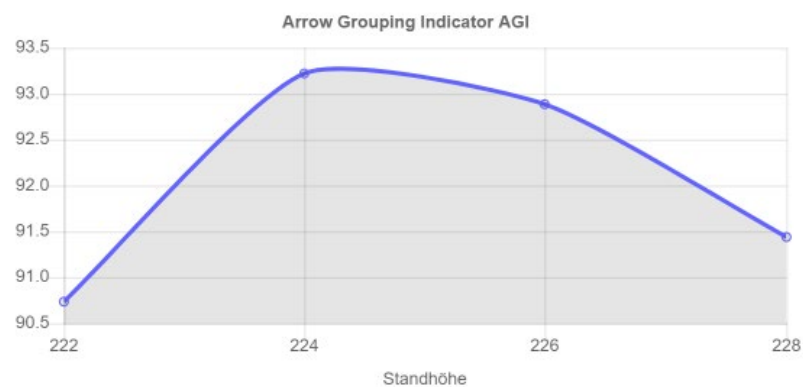
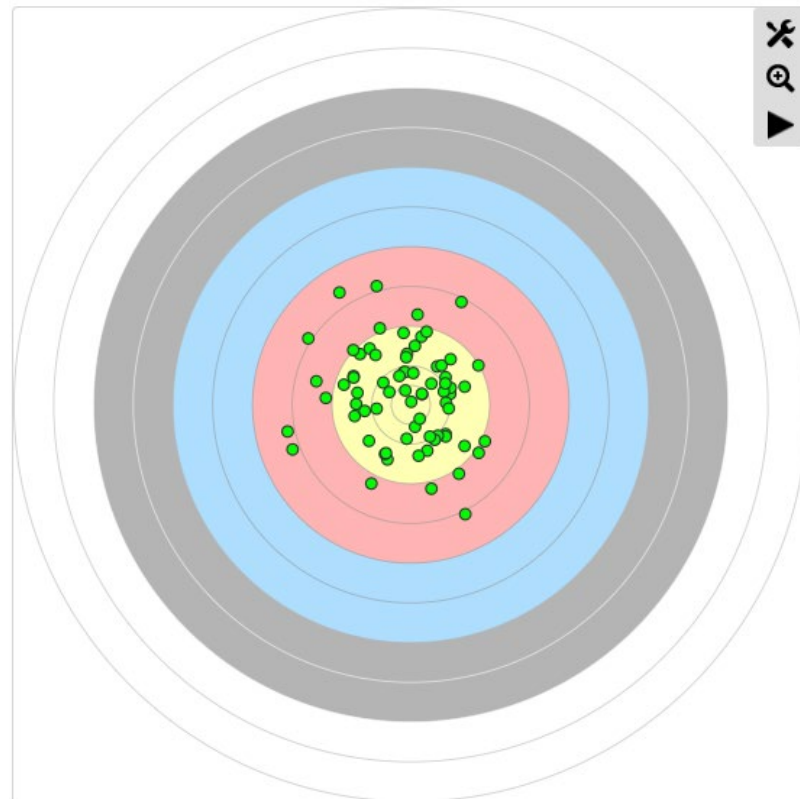


Figure 11: AGI – Brace height

In the example above, it might be the case that the bow sounds „good“ in the range of 223 – 227 mm, however the optimum in this respect usually cannot be found with the acoustic method.

Bear in mind: The higher the amount of arrows shot per brace height, the more significant the result will be.

c. Nocking Point

A generic challenge of any bow is that its force center is located where the hand backs the riser but the arrow rests 4 -5 cm on top of it. This is the reason why it is not a good idea to nock in the arrow exactly in the middle of the string. It would fly up into the sky in an unpredictable way.

But if you nock in the arrow above the middle of the string then the rear end of the arrow is not necessarily getting accelerated on a straight line ahead, unless you correct the upper and lower draw of the bow.

This is – in short – the problem which is supposed to be solved by an asymmetric tiller and by appropriate nocking point height. The intention with this setting is that the accelerating force proceeds exactly in the horizontal layer of the axis of the arrow.

In case of incorrectly set parameters, after releasing the arrow will meet a vertical inclination versus its trajectory and thus during flight will swing up and down vertically around its balance point (called „porpoising“). Due to its aerodynamics, this movement will affect fletched and unfletched arrows differently so that these arrows hit the target in different heights.

The criterion for correctly set nocking point heights is therefore that bare shafts (= unfletched arrows) hit the target on the same height as fletched ones. This is the bare shaft test which is described in most books related to bow tuning.

The RyngDyng® App supports these bare shaft tests on the one hand by marking individual arrows as bare shafts and on the other hand when plotting the overlay of many arrows, a scatter ellipse for all bare shafts is computed and shown separately from the scatter ellipse of all fletched arrows. In this way, we get a high statistical significance for the group center of the bare shafts relative to the group center of the fletched ones.

The subsequent images display potential results of bare shaft tests. They illustrate the overlay of four ends with six arrows each, of which one was a bare shaft. The bare shafts are marked as blue dots. On the left side, you can see that the group center of the bare shafts tend to be lower than is the group center of the fletched arrows. On the right side, the result is the opposite. On the left, the nocking point is too high, whereas on the right it is too low.

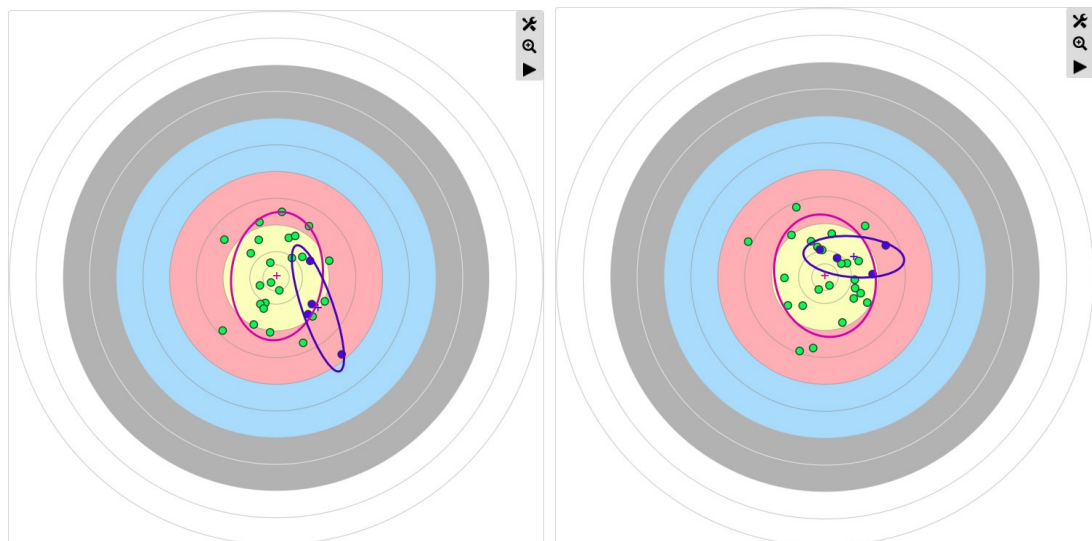


Figure 12: Bare Shaft Test for Nocking Point

You can imagine now that the position of the bare shafts relative to the group center and covering various ends wouldn't have been easily to spot without recording the exact positions. But when overlaying the four ends and applying statistical assessment as described, this information can be gathered easily and with a high significance.

d. Tiller

The determination of the correct nocking point height as explained in chapter c has been made with the default setting for the tiller difference in the range of 4 - 6 mm. You can however, also identify the optimal value for the tiller by means of a grouping test.

Step by step you can proceed testing analogously according to section *b* in the range of 0-8 mm. This needs to be conducted by adapting the tiller difference in steps of 1 mm each. In order to not change the nocking point height with every change of the tiller difference, you would need to measure the nocking point height and take it back to the already gathered optimal value, before you shoot arrows.

For any set value an arbitrary number of arrows from 30 m distance are now getting recorded. As with every grouping test, aim at the same targeting point - no matter where the arrows hit.

The RyngDyng® App also supports this grouping test by storing the respective tiller setting together with the corresponding arrows. For every tiller setting the grouping value AGI is computed for all arrows referring to this tiller setting.

The result will then be presented in a clear and precise matter:



Figure 13: AGI – Tiller

It becomes apparent that in this example a tiller difference of 3 mm holds the best arrow grouping.

e. Matching of Arrow with Bow

The selection and matching of the arrows with the bow is a science of itself. In summary, it is about adapting the dynamic behavior of the arrow (i.e. when compressed by acceleration and with subsequent oscillations) to the dynamics of the bow when releasing.

During the acceleration phase 'in the bow', the arrow shall make exactly $1 \frac{1}{4}$ oscillations so that in the moment when the fletches of the arrow pass by the riser window the arrow tail swings outwards. Like that the arrow bends around the riser window without touching it. This is called the archer's paradox.

The frequency of the oscillation of an arrow is - depending on its weight and stiffness – at about 50-90 Hz. $1 \frac{1}{4}$ oscillations therefore take about 13-25 milliseconds. This time period needs to correspond exactly with the acceleration time in the bow until the arrow fletches pass the riser window. During that time span, the arrow gets accelerated up to 50-70 m/s (depending on bow and arrow).

The oscillation frequency of an arrow can be influenced by

- Stiffness of the shaft (spine)
- Length of the shaft
- Mass / material of the shaft
- Weight of the arrow head

- Weight of the fletches, wrap and nock

The acceleration time in the bow can be affected by

- Draw weight
- ‚Speed‘ of the limbs (= its masses and draw-force curve)
- Weight of the string with nocking point
- Brace height.

Further parameters have an influence, too. E.g. the release procedure at the bow string has consequences as it affects the string on the first two centimeters. (see section 5.b).

Besides the matching of oscillation frequency and acceleration time, you need to ensure that the arrow wouldn't get pushed sideward out of its ideal trajectory when being compressed during acceleration. This is what the parameters of the button are for: center in rest position and strength of the button spring.

A mismatch of arrow and bow can happen in two different ways:

First, it may happen as with a false nocking point height that the arrow right after shooting flies in a certain angle to its trajectory and therefore swings (fishtailing) during the flight. This fault may also appear or at least be intensified by a false center position of the arrow or a false strength of the button spring.

Second, several clearance problems may occur, where the arrow when shooting touches with some parts the bow (button pin, arrow rest, clicker). As a result, the arrow tends to swing with increased frequency (called 'minnowing').

In any case, the grouping values tend to get worse and the bare shafts behave differently than the fletched arrows. This is where our statistical analysis will support you with diagnosing and tuning.

For a proper matching of the arrows we recommend to determine the spine of the shaft according to a selection table for arrows, usually provided by manufacturers, and to start with a shaft which is initially about 1 inch too long. Alternatively, you can use Archery Analytics arrow matching tool:

https://www.archery-analytics.com/en/public/support/arrow_frequency

Such a too „soft“ arrow (= too low oscillation frequency) you can cut step by step until it matches. This approach allows you to keep the desired draw weight, the head weight and the fletching of the arrow and still achieve a good match with the bow.

As the button has a similar effect on the arrow's trajectory as its frequency, we advise to do the tuning step by step.

You can start with the basic setting of the button as described in chapter a. By means of the bare shaft test, the shaft is getting shortened gradually until the arrow is just slightly „too soft“. Then, do a more precise tuning of the button according to section g. Afterwards the bare shaft test is resumed again and if required you can shorten the arrows further. This can be pursued until the bare shafts are at the same horizontal position as the fletched arrows. The advantage of this stepwise approach is to avoid the shortening of a supposedly too soft arrow whereas the problem was caused by a false button setting.

We advise experienced archers to conduct the bare shaft tests over a distance of 30 m; or over at least 15-20 m.

In the illustrations below you can see the bare shafts before and after the shortening of the shaft by 5 mm. As a right hand bow was used, the bare shafts hit too far on the right and indicate they are too soft. After they were shortened again by another 5 mm the shafts became stiffer, hence the bare shafts shifted more inwards (right image). This approach can be continued until the group center of the bare shafts correspond with the group center of the fletched arrows.

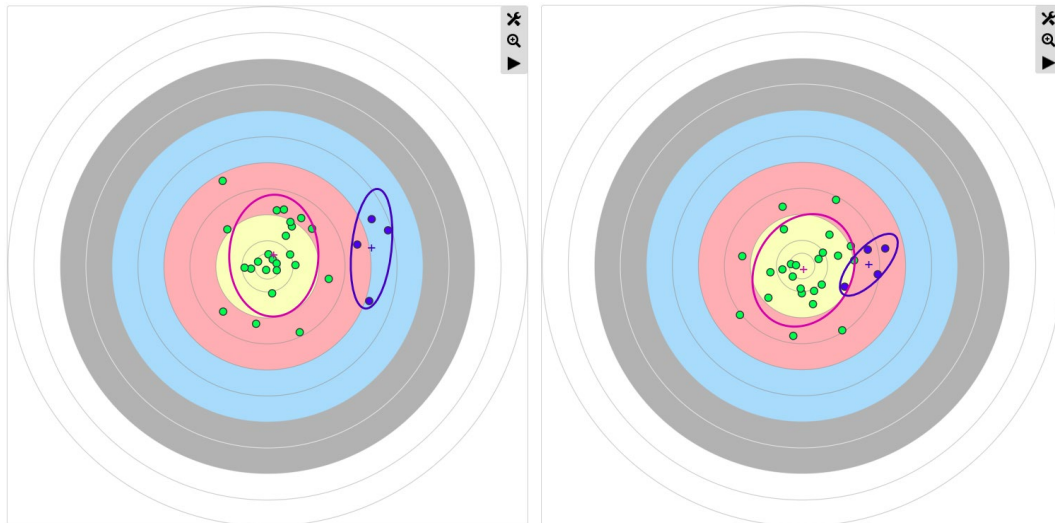


Figure 14: Bare Shaft Test for Arrow Length

Also here it applies that the significance of the group centers is increased by overlaying several ends. If you intend to do this without RyngDyng[®] and with single ends only, it may happen that you cut the arrow shaft again without actually having the need to do so.

If it is not possible to shorten further - be it because of the draw length or clicker position - the arrow can be made stiffer by reducing the weight of the arrow head or increasing the draw weight. (Though not recommended, as you can read further below in the document).

Limited possibilities exist to correct when an arrow is too stiff (= too high oscillation frequency).

You can

- Increase the head weight
- Increase the draw weight of the bow
- „Speed up“ the bow by a lighter string
- Reduce the spring strength of the button (if it is not too weak yet)

These modifications however face practical limitations as they are usually chosen for other reasons than the tuning of the oscillation frequency of the arrow. In the worst case you would need to purchase other shafts with a higher spine value.

As soon as the arrow is well tuned you can measure and note down its oscillation frequency (see chapter 8.a *Oscillation Frequency of Arrows*). By the way, this can be done with the entire set of arrows. In addition to the same weight of all arrows this parameter is essential for a well-adjusted set of identical arrows.

f. Clearance

A clearance problem occurs, when the arrow tail doesn't pass smoothly the riser window but touches it instead. This is a problem that needs to be solved; otherwise you wouldn't be able to achieve a good enough grouping indicator (AGI value).

Hints as how to recognize a clearance problem can be found in literature, e.g. when minnowing is being observed, a fast „wiggling“ of the arrow on the first meters after releasing. Another idea is to spray the arrow tail with some powder color and check after shooting whether abrasions occurred.

With RyngDyng[®], you can recognize clearance problems when analyzing the AGI value. This is because clearance problems tend to have a stronger effect on shorter distances than on larger ones. And - as you know in the meantime - the AGI value is computed independently of the distance; you would just need to compare the AGI of arrow groups of 10 m distance with the ones of e.g. 40 m distance. If it indicates then the AGI on the short distance is worse (e.g. more than 5%) you can conclude a clearance problem.

Solutions to counter clearance problems are:

- Slightly twisting the nock to bring the fledges to a different position
- Lowering the height of the fletching
- Placing the center position of the arrow further outwards

Has the clearance problem been solved in this way, the arrow matching should then be repeated according to section e.

g. Button

The static center position of the arrow is set by the button by turning the button into the riser. The elastic button pin enables the arrow to slightly shift its position inwards. The intention with this configuration is to keep the arrow exactly on the line towards the target point, despite its compression and therefore oscillating behavior. This is called the 'dynamic center position' of the arrow.

Since this static and dynamic center position of the arrow has similar effects on its trajectory as its oscillation frequency respectively the acceleration time in the bow, we advise you to not do the button settings by means of the bare shaft test. Otherwise it may happen that the arrow has a false oscillation frequency and consequently could get „corrected“ by an also false button setting. The bare shaft test results then would appear to be good although the entire system is not adjusted correctly and high AGI values wouldn't get achieved.

Even clearance problems may occur due to this mismatch, even though the bare shafts don't indicate it.

The best method for the tuning of the button is the walk-back test according to Berger.

In the common illustrations of the walk-back test, people assume that from various distances a fletched arrow is getting shot. So when choosing this approach a problem with the insufficient statistical significance exists. This is the reason why Archery Analytics developed the *Statistical Walk-Back Test*. The assessment is based on the statistical analysis of an arbitrary amount of arrows per distance.

Now to begin with, a cross as the target mark is fixed in the upper part of the target. In our version of the Walk-Back Test it is a blue cross which RyngDyng[®] is able to recognize.

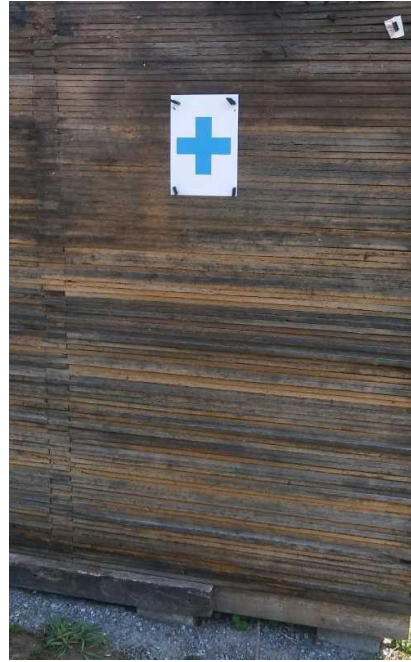


Figure 15: Blue Target Cross for the Statistical Walk-Back Test

Set the sight to approximately 15 m. Afterwards you can shoot an arbitrary number of arrows from various distances, such as 5 m, 10 m, 15 m, 20 m, 25 m, 30 m, 40 m. Please make sure to always keep the targetting point fixed to the target cross. It is sufficient to shoot one or only a few arrows when the distance is short, for larger distances more arrows should get shot as it increases the reliability of the test. The test is finished when you notice the arrows stick only in the lower part of the target. In the RyngDyng® App, the result of the Statistical Walk-Back Test looks as follows:

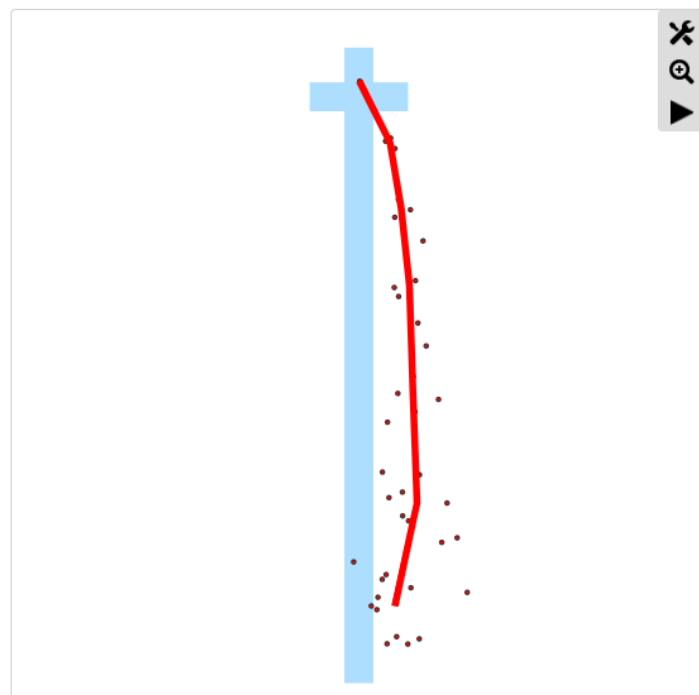


Figure 16: Walk-Back Test Result 1

The upper arrows are those of the shorter distances. As you can see, fewer arrows per distance were shot. The further down the arrows are the larger the distance was. Consequently more arrows were shot when the distances were larger.

A curve to the right as referred to in Figure 13, which tends to return towards the middle, suggests a false center position of the resting arrow. When using a right hand bow you need to bring the arrow slightly outwards whereas with a left hand bow more inwards. The button position shall only get changed by 1/8 – 1/4 turns before you can test again.

Imagine the red line would be curved towards the left, it would then be opposite: With the right hand bow the arrow needs to be put more inwards whereas with the left hand bow it is outwards.

The following picture shows another result of the Walk-Back Test:

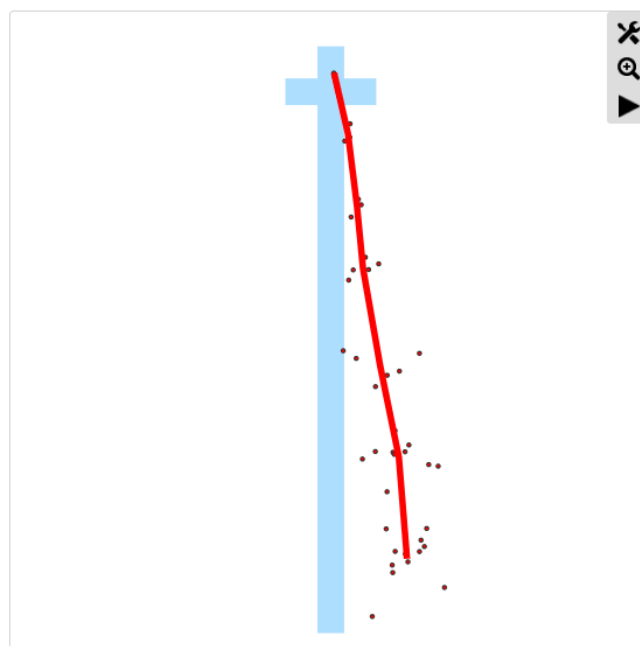


Figure 17: Walk-Back Test Result 2

A line sloped to the bottom right corresponds to a wrong choice of the strength of the spring of the button. When using a right hand bow, the spring needs to be harder, with a left hand bow it needs to be softer.

If the line slopes to the bottom left, the spring needs to be softened when it is about a right hand bow and when it is a left hand bow it needs to be hardened.

Both defects – the false center position and the false hardness of the spring – can overlay. This can be observed here:

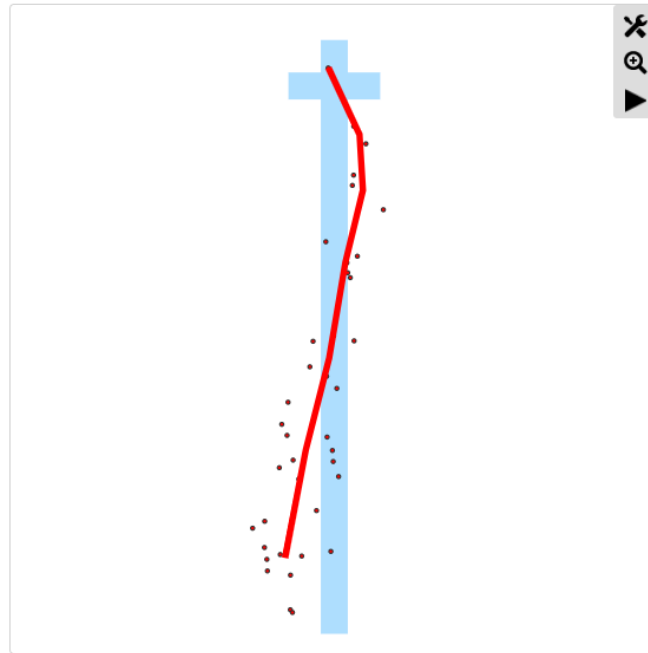


Figure 18: Walk-Back Test Result 3

The red line initially proceeds to the right before it finally turns to the bottom left. It indicates both a false center position as well as false hardness of the spring. In order to avoid too many adaptations of the settings at the same time, we advise in the case of a right hand bow to soften the spring followed by a repetition of the Walk-Back Test (with a left hand bow it is reversed, i.e. to harden the spring).

When both the resting center position of the arrow as well as the hardness of the spring is correct, the result of the Walk-Back Test looks as follows:

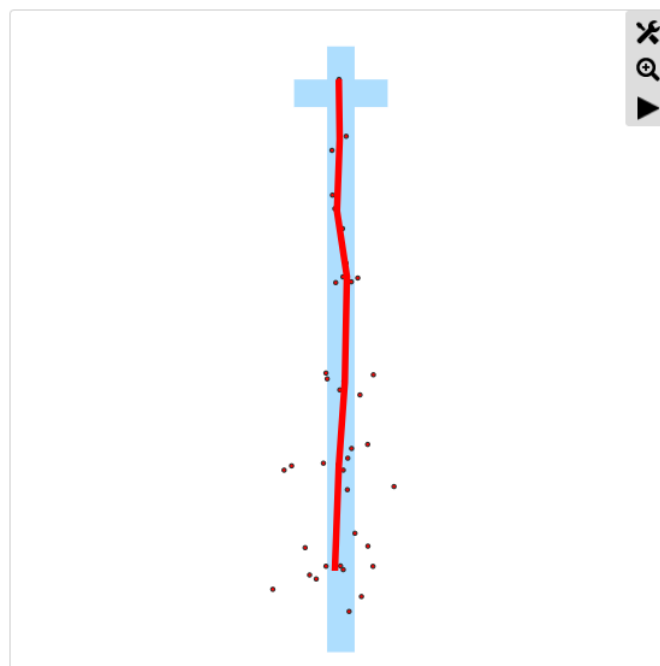


Figure 19: Walk-Back Test Result ok

The image clearly illustrate that the execution of the Walk-Back Test with only one arrow per distance can easily lead to misdiagnosis. You can therefore only recognize if the button is correctly set when you do a statistical computation of the red lines.

h. Fine Tuning

If you adjusted the settings according to the before mentioned sections a – g, the optimization can now be carried out. For this purpose only grouping tests respectively optimizations of the normalized dispersion are being conducted.

We begin with the exact measuring of the optimal brace height. This is done by taking the already identified brace height and varying it upwards and downwards in small steps of 0.5 mm. In order to reach a sufficient significance, we recommend to shoot plenty of arrows (e.g. 24) per brace height. The brace height with the highest AGI will be kept.

The nocking point height is determined accordingly, though the focus is on the normalized vertical dispersion. The nock point height is varied in small steps of 0.5 mm upwards and downwards respectively, the normalized vertical dispersion is measured and minimized.

If we would now conduct a bare shaft test with the so-computed optimal nocking point height, in most cases we would realize the bare shafts being positioned too deep. However this is acceptable if the vertically normalized dispersion becomes better.

The third step of fine tuning refers to the hardness of the spring of the button pin. Focus is on the horizontal normalized dispersion. You can adapt the hardness of the button spring in very small steps in both directions and by that that determine the minimal horizontal dispersion. Also here, the bare shaft test could possibly indicate somewhat too hard arrows. Again the better value for the horizontal dispersion is decisive here.

Other means of fine tuning are related to the fletching. You can vary the angle or the position of the fletching and check for the best AGI on differing distances. However, please don't change significantly the already optimized matching of oscillation frequency of the arrow. In case you have changed the position of the fletching, double-check the oscillation frequency of the arrow using the method described in chapter 8.a.

5. Common Shooting Faults

The following applications demonstrate how common faults in the shooting technique have an impact on statistical metrics. These analysis however presume a properly configured material. If not, the configuration defects overlay with shooting faults and hence a clear separation becomes difficult.

Especially when it comes to the analysis of shooting faults it is of utmost importance to include a sufficient amount of arrows in the analysis. You know, this allows for high significance.

a. Unstable Anchor

A (vertically) instable anchor at the chin implies that the rear end of the arrow has no constant vertical distance to the aiming eye. As an example, this can be caused by a more or less pressing of the hand at the chin or the cheekbone, or by the fact that teeth when shooting are not always exactly on top of each other. Also the shape of the drawing hand (e.g. the position of the thumb) can cause an instable anchor.

Such an instable anchor alters from shot to shot the vertical angle of the arrow when shooting. This is similar to varying the sight by height. The consequence is an increased vertical dispersion.

A characteristic of such a fault is that the distance-independent *normalized vertical dispersion* doesn't or only rarely depends on the distance (see section 3.d). Because this fault is primarily an inaccuracy of the vertical shooting angle and the normalized vertical dispersion is a measure for that angle inaccuracy.

So when you realize a too high vertical dispersion (e.g. higher than a ring width, depending on your personal claim) and at the same time the normalized vertical dispersion in the range of 18 – 40 m is independent of the distance, then proceed by checking whether the anchor is stable or not.

b. Release Faults

Release faults have (at least) two dimensions:

The release procedure can sometimes be faster, sometimes slower. For instance, by succeeding with the spontaneous relaxing of the finger muscles, or when the finger open rather consciously. The result is an uneven launch speed of the arrow. Because if you release the fingers in an „un-relaxed“ way, the draw length first is getting reduced a bit before the string is actually released. Hence, less energy is transferred to the arrow compared to when releasing spontaneously.

Furthermore, the lateral impulse on the string – which is unavoidable with every finger release (tab, glove) - can once be higher or lower. This is revealed by a horizontal dispersion of the arrow positions. Of course, both faults can overlay.

The first fault with the uneven launch speed of the arrow is relatively easy to diagnose: such an uneven shooting speed has a bigger effect on larger distances than on shorter ones. This is because a highly arched trajectory is more sensitive to fluctuations in the shooting speed than a largely straight trajectory on a short distance (for details see chapter 8.b). It follows that with this kind of fault the *normalized vertical dispersion* at larger distances is higher as with short distances. So, if you discover this correlation in the statistical analysis you better look closer at the release procedure with regards to the relaxing of the fingers.

The horizontal dispersion however becomes bigger if the lateral impulse on the string varies. Since there may be several explanations for horizontal dispersion, the unambiguous diagnosis is not that simple.

There is the possibility to compare fletched arrows with bare shafts. This is because the unbalanced lateral impulse affects the arrows after shooting in such a way that there is a horizontal angle of the arrow to the trajectory. This has a similar effect as with a mismatched oscillation frequency of the arrow.

The lateral angle of the arrow to the trajectory is corrected faster due to its fletching than it is with the bare shaft. Thereby the bare shaft receives a bigger lateral drift. This drift however can sometimes be stronger and sometimes be weaker depending on the variation of the lateral impulse when releasing.

As a conclusion, if you shoot a high quantity of fletched and unfletched arrows, the bare shafts will indicate a higher horizontal dispersion with this kind of release mistake than the fletched arrows.

The following image shows each 18 fletched (green) and unfletched arrows (blue), from a distance of 18 m on a 40s target face. The scatter ellipse of the bare shafts has a larger horizontal width. Hence the finding is a varying lateral impulse on the string when releasing.

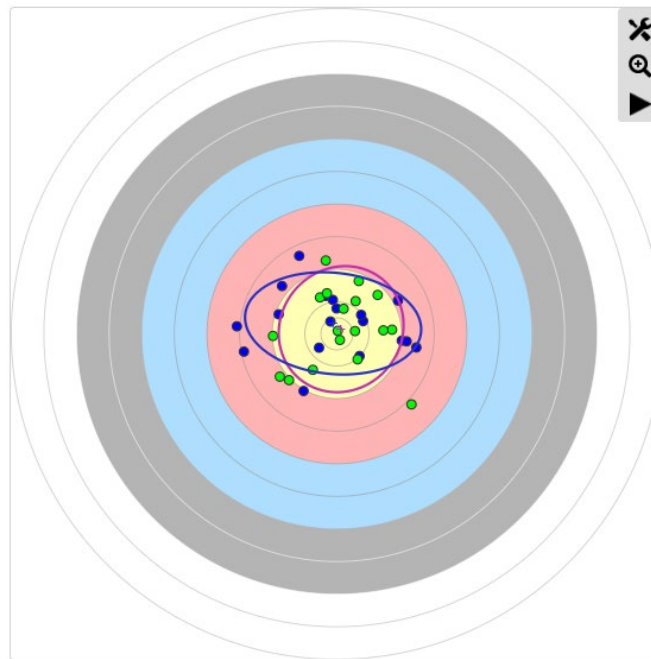


Figure 20: Horizontal Dispersion of Bare Shafts

c. Unbalanced Finger Pull

When doing finger-release usually the string is held by three fingers and the nock of the arrow is between the upper two fingers. This is why the lower, third finger may have a distracting effect on the string.

On the one hand, if you pull the string stronger with the lower fingers, the strength with which you hold the string changes whereas the draw length remains the same. Some more energy is stored in the drawn bow and this energy is transmitted to the arrow. Particularly the lower limb will therefore be more tightened.

On the other hand, compared to a decreased draw force in the lower fingers, the release will proceed differently as the string will receive an additional lateral impulse caused by the higher tension in the lower fingers.

With this combination, the arrow receives more energy and hence a higher launch speed and at the same time a slightly bigger lateral impulse.

If now the strength of the lower finger release varies from shot to shot then this will lead to an overlay of a higher vertical dispersion with a higher horizontal one. This can be seen in the subsequent illustration of a group consisting of 36 arrows of a right handed archer.

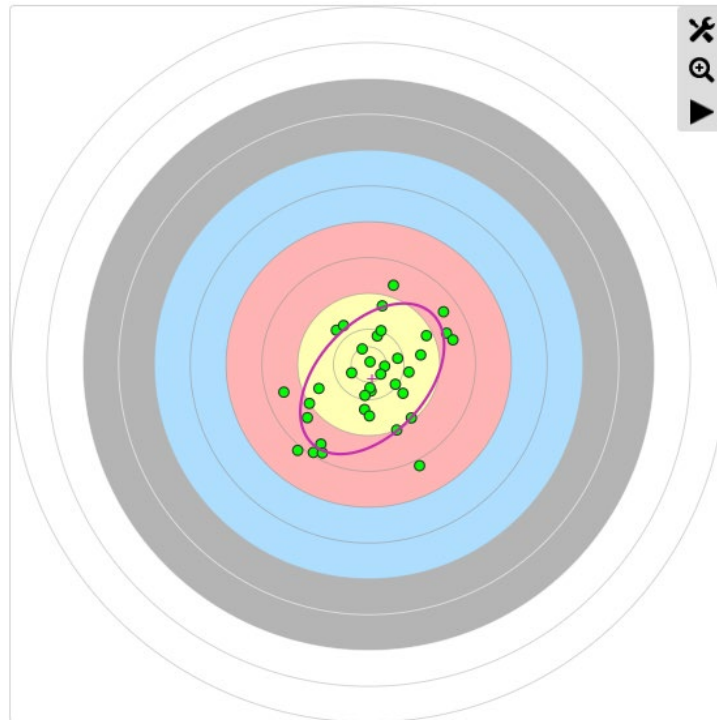


Figure 21: Unbalanced Finger Release

The stronger the lower finger draw the higher the speed of the arrows and consequently the higher they hit the target. At the same time when releasing, the lateral impulse on the string becomes bigger and the arrows when shooting take a slight angle to the right. The result is an inclined scatter ellipse marking the course of the arrow group from the bottom left to the top right.

To recognize this fault in the statistical analysis with a high reliability, you should have many arrows represented as a group and have them overlaid by several ends. The shooting distance should be at least 50 m so that the vertical dispersion becomes clearly visible.

d. Inconsistent Bow String Shadow

The string shadow in archery is what is called „notch“ when it comes to guns shooting. However the string is perceived only in a diffuse way because of its short distance to the aiming eye. This is why it is called „string shadow“. For almost all archers, it is a challenge to get this shadow into the exact same position relative to the sight tunnel.

If now the position of the string shadow varies from shot to shot, this would correspond to a variation of the horizontal shooting angle. Similarly to when you change the sight tunnel laterally from shot to shot.

This mistake can be easily recognized as the *normalized horizontal dispersion* is independent of the shooting distance. This dispersion expresses precisely such an inaccuracy of the angle. So if you intend to diminish the horizontal dispersion and find out that it is distance-independent, we recommend you to work on the constant position of the bow string shadow.

Another mistake relating to the bow string shadow are changing lighting conditions. This is because the perception of the bow string shadow heavily depends on the incidence of light.

If the light conditions change, e.g. due to a change from sun to shadow or clouds, or the illumination of an indoor shooting range, many archers will notice a lateral drifting off the arrow group. If the light conditions don't change from arrow to arrow, the horizontal

dispersion will not change at all or only slightly, but instead only once or more in the long-term.

You can identify the problem by analyzing the group center from end to end over a longer period and so you will be able to recognize the lateral drifting of of the group center. The horizontal dispersion shouldn't change from end to end in this situation.

The RyngDyng® App supports the analysis of the light conditions by storing both the shooting direction per end and the exact time of each arrow. Like this the information in which angle the sun was at every end can be easily retrieved.

You can further use the given free text field and use it for remarks and notes regarding the light conditions. The information is then available for later analysis.

e. Varying Pressure Point at Grip

Caused by an inconstant posture of the bow hand, the point of highest pressure between hand and grip will not always be at the same spot. One time it will be higher positioned, another time it will be lower.

A too high or too low position of the pressure point when shooting has a skidding effect. The impact of this unbalanced pressure point on the bow resembles an alteration of the tiller. If the pressure point is too high then the upper limb is getting more tension. If it is too low, then the lower limb is tenser.

An inconstant pressure point has the same effect as if the tiller would get slightly changed from shot to shot and hence the nocking point height. This fault can thus be recognized as it affects bare shafts differently than fletched arrows (see section 4.c.). The consequence will be a higher vertical dispersion of the bare shafts compared to the fletched shafts.

The image below shows each 18 fletched (green) and unfletched (blue) arrows, shot from 18m distance onto a 40s target face. The scatter ellipse of the bare shafts marked in blue holds a higher height than the scatter ellipse of the fletched arrows. This is why the finding here is an unbalanced pressure point at the grip.

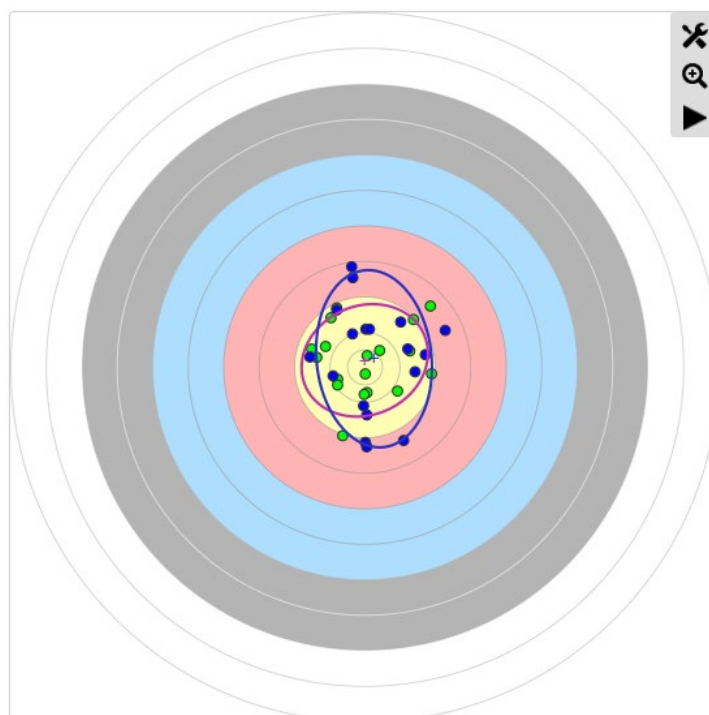


Figure 22: Unbalanced Pressure Point at the Grip

f. Insufficient Body Tension

In the final stage of the shooting procedure (= clicker end phase), a multitude of muscle groups need to be controlled. These muscles span over the entire body, perceived subjectively by archers as wide-ranging body tension.

Depending on the condition, archers succeed more or less well in sustaining their body tension. Many archers will experience a weakening, particularly after they shot a high amount of arrows. If the body tension diminishes, no matter if it is caused by inertia or decreasing concentration, it certainly affects the shooting performance.

Even if for every shot the arrow is pulled through the clicker, the arrow – due to insufficient body tension - will get less speed when shooting as with appropriate body tension.

It can often be observed that immediately after „clicking“ but still before the actual release, a relaxing of involved muscle groups begins and hence the draw length diminishes. This may happen due to too less counter pressure in the bow arm or due to decreasing tension in back and shoulder.

The result of a reduced arrow speed is a too deep hit on the target face, especially when it comes to bigger distances from 60m onwards. If the body tension varies from shot to shot by defeating fatigue, the vertical position of the arrow tends to fluctuate.

An easing of the body tension manifests itself in the arrow groups on the one hand by having the group centers „moving“ downwards with an increasing number of arrows. On the other hand, at the same time an increase of the vertical dispersion may happen, this being the case when due to mental focus the tension can be revived sometimes. The larger the shooting distance, the more explicit these effects will appear.

The following series of arrow groups overlaying several ends from a 122s target face on 70 m distance shows the effect of a decreasing body tension from left to right:

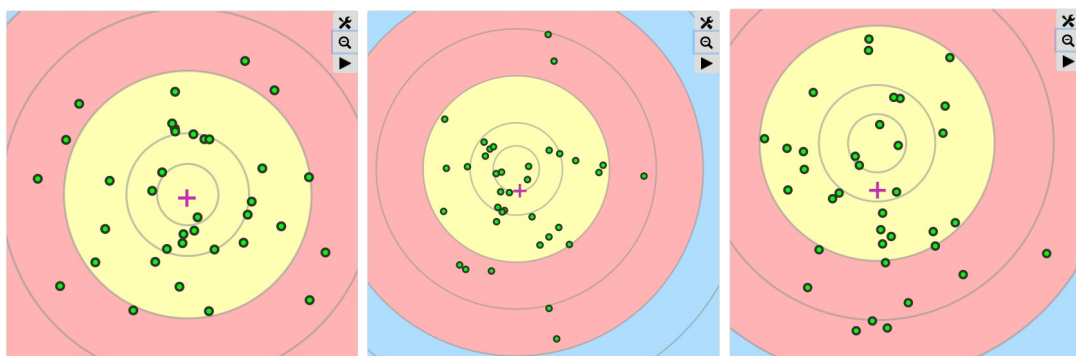


Figure 23: Easing of Body Tension

The vertical shifting and dispersion of these arrow groups are

	Left	Middle	Right
Vertical Shifting	- 0.4 cm	-2.9 cm	-4.9 cm
Vertical Dispersion	13.2 cm	15.7 cm	16.4 cm

In addition to the vertical shift and dispersion, also the horizontal dispersion can increase. This can be the case if e.g. a further mistake may appear as described in section b *Release* or *Inconstant Bow String Shadow*, which is caused by relenting concentration.

Because of the overlaying of the past three to five ends, it can be very early and reliably noted that the vertical shifting and/or dispersion changes. This information is by far more reliable than the information gathered by observing individual ends only.

Due to continuous assessment of the past ends a kind of early warning system can be established even before the problem becomes really apparent.

Also, if you wish to analyze competition data after the tournament, you are able to determine in which phase of the competition the first signs of an easing of the body tension showed up.

6. Special Tests

a. AGI at Increasing Distance

The *Arrow Grouping Indicator AGI* (see section 3.f) is a metric expressing the quality of the arrow group independent of the shooting distance. Reversely, this means that if this metric is dependent on the distance you can draw significant conclusions.

To conduct this test, shoot from 10, 20, ..., 90 m distance (respectively until the maximum distance for your competition class is reached) at least 30 arrows each. Afterwards compare the AGI values of these distances. If you realize that the AGI value is on 10 m lower than on 20 or 30 m, a clearing problem might be the cause (see section 4.f).

If you however notice the AGI value is decreasing on a larger distance, then two more cases need to be distinguished.

Case 1: Both the horizontal normalized dispersion as well as the vertical normalized dispersion have increased. The scatter ellipse thus has the same rather round shape as it has on short distances.

Case 2: Only the vertical normalized dispersion has increased but not the horizontal one. So the scatter ellipse is getting stretched vertically.

In the first case it is about a general, disproportionate increase of the dispersion at large distances. The most likely cause is a too high spin of the arrows especially in the last part of the trajectory. In this case we recommend to reduce the angle of the fletches and do the test again.

In the second case, the vertical dispersion increases disproportionately. While at very large distances you will often have a slightly higher normalized dispersion as the trajectory of the arrow becomes higher. Inaccuracies while releasing or based on easing body tension have a bigger impact at arched trajectories as with the almost straight trajectories on short distances.

If you are somewhat satisfied with your vertical dispersion on short distances, but not with the vertical dispersion on large distances, then try to increase the draw weight and hence keep the trajectory lower. Unfortunately, a re-tuning of the arrows as explained in section 4.e is required then. As the arrows need to be made stiffer at a higher draw weight, there is also the possibility to achieve it by shortening the shaft, provided the arrows have some spare left in length. Under no circumstance you should try to make the too soft arrow stiffer by the button. Please see chapter 4.g.

7. How to select a well matched set of arrows

Even if the arrows are well matched to the bow, small differences between the individual arrows remain. Especially for competitions, it is important to select the best 6 arrows from your 12 tuned arrows in order to achieve a good grouping, i.e. a high AGI.

However, before you start shooting and measuring the AGI values for individual arrows, you should make a pre-selection based on basic measurements:

1. Same weight
2. Same natural arrow frequency

With a fine scale, you can measure the weight to an accuracy of 0.1 grain and adjust the weight of individual arrows if necessary. For example, by adding more or less glue to the tip, or more or less fixing tape to the fletching.

However, this also changes the mass distribution and thus the natural oscillation frequency of the arrow (see Appendix 8.a or https://www.archery-analytics.com/en/public/support/arrow_frequency)

On the one hand, this frequency must have a certain value in order for the arrow to match the bow. Second, all arrows should have the same frequency so that they can form a narrow group. The reason for this is easy to understand:

Immediately after releasing, the arrowhead swings back and forth, depending on the deflection of the shaft by about 2 cm (1 in). While traveling a distance of 18 m (20 yards), the arrow then makes about 18 full swings. If an arrow arrives in such a way that the tip swings straight to the left when it hits, but the following arrow swings to the right when it hits, then these two arrows have a difference of 2 cm (1 in) at the target - only due to the different phase of the natural oscillation!

If you calculate this further, you will find that all arrows should have a maximum deviation of 0.1 Hz if you want to keep the arrow oscillation 'in phase' at 18 m (20 yards).

Therefore, an exact measurement of the natural arrow frequency according to appendix 8.a is required.

If you want to adjust the weight of an arrow, the following rule applies: More mass at the tip reduces the natural frequency only slightly, more mass at the fletching / nock reduces the natural frequency significantly. Reducing the masses increases the frequency.

You should to keep both values in mind when tuning individual arrows: the change in mass and the simultaneous change in natural frequency. The tool https://www.archery-analytics.com/en/public/support/arrow_frequency can help you to calculate the change in frequency when you want to change the arrow mass at the tip or at the rear end.

8. Appendix

a. Oscillation Frequency of Arrows

For the matching of an arrow with the bow it is essential that the oscillation frequency of the arrows fits with the acceleration time in the bow. The arrow then should ideally make $1 \frac{1}{4}$ turns until the fletches pass the riser window. Thereby you can assure the maximal free space when passing by.

You can estimate the likely required oscillation frequency by means of some bow and arrow parameters, which are:

- dl : The effective draw length in m (= the length of how far the string is getting pulled backwards = distance of arrow tip to the edge of the clicker while the arrow is nocked in)
- dw : The draw weight at full draw in Newton (= draw weight in lbs \cdot 0.454 \cdot 9.81)
- bh : The brace height in m
- fd : The distance of the highest point of the fletching from the bottom of the nock in m
- am : The mass of the arrow in kg

With these parameters, you can calculate an approximate oscillation frequency of the arrow of

$$f = \frac{1.25 \cdot \sqrt{dl \cdot dw \cdot 0.8}}{\left(\frac{\pi}{2} \cdot dl + bh - fd\right) \cdot \sqrt{am}}$$

Example:

- $dl = 47 \text{ cm} = 0.47 \text{ m}$
- $dw = 34 \text{ lbs} = 34 \cdot 0.454 \cdot 9.81 = 151.4 \text{ N}$
- $bh = 230 \text{ mm} = 0.23 \text{ m}$
- $fd = 45 \text{ mm} = 0.045 \text{ m}$
- $am = 19 \text{ gramm} = 0.019 \text{ kg}$

yields an oscillation frequency of $f = 74 \text{ Hz}$.

This set value can now be compared with the actual frequency of the arrow which is being determined as follows: use an app intended to tune musical instruments (e.g. „gStrings“) and measure the oscillation frequency by slinging the arrow.

For that purpose, just hold the arrow loosely between two fingers at approximately 8 cm below the fletching and let it hang down with the tip ahead. Then tap the arrow with its bottom third against the edge of a table.

You can now vary the position of the fingers slightly up and down until the arrow swings most. You have now found the so-called oscillation nodal point.

Next take the arrow, tap it a bit harder and hold the middle of the shaft directly in front of the microphone of your mobile with the before mentioned tuning app running. It shall now reveal the oscillation frequency of the arrow.

In this manner you can detect whether or not the arrow has the expected frequency. In case of a too low frequency the arrow seems to be too soft, in case of a too high frequency the arrow is too stiff.

However, please note that this approach only gives an indication. The formula above for the calculation of the necessary frequency is just an approximate solution for a linear force-deflection-curve of the bow. A more precise calculation for the actual measured force-deflection-curve requires a computer.

A more accurate calculation for an actually measured force-displacement curve requires the help of a computer. We implemented this frequency calculation on our server and made it available to all interested archers using this link:

https://www.archery-analytics.com/de/public/support/arrow_frequency . There you can also test whether the arrow fits the bow and which measures lead to a better match.

By the way, with the above given parameters, you can also obtain an estimation of the arrow speed in m/s:

$$v = \sqrt{\frac{dl \cdot dw \cdot 0.8}{am}}$$

For our example, it means $v = 54.7 \frac{m}{s} = 197 \frac{km}{h}$. The factor 0.8 in the formula corresponds to the typical degree of efficiency of a recurve bow, which is 80%. This is the share of energy contained in the arrow, whereas 20% remain in the bow.

The formula for oscillation frequency can also be depicted in easier terms – by using the speed v :

$$f = \frac{1.25 \cdot v}{(\frac{\pi}{2} \cdot dl + bh - fd)}$$

b. Normalized vertical dispersion on large distances

In this chapter we will discuss on how the *normalizes vertical dispersion* varies between short and large distances.

The used mathematical model for calculating the trajectories also incorporates the air drag of the arrow, which is dependent on the velocity and on the oscillation of the arrow shaft. The oscillation will lead to a greater area that is exposed to the wind. This effect increases the so-called C_w value, which normally is a constant value when calculating the air drag. As the oscillation diminishes exponentially while the arrow is flying, the air drag decreases as well. As a result, the calculated trajectory has the form of a distorted parabola, being steeper towards the target.

Let's now have a look at two calculated trajectories over a distance of 70 m. The difference between the two is a difference in the initial speed of 3 km/h = 0.833 m/s. The slower arrow therefore has got a diminished kinetic energy by 2.77%. All other parameters are kept equal.

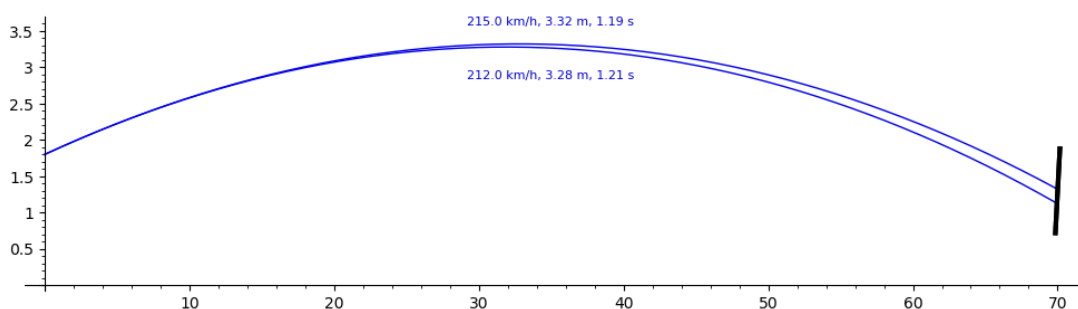


Figure 24: Two trajectories over 70 m distance

The next picture shows the situation at the target in more detail:

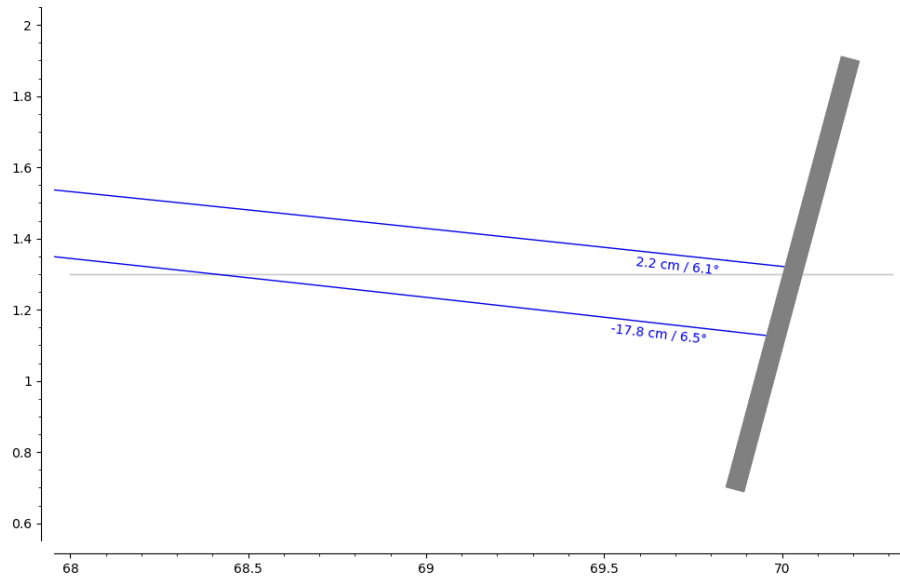


Figure 25: Target at 70m

The vertical distance between the two arrows on the target is $2.2\text{ cm} + 17.8\text{ cm} = 20.0\text{ cm}$. The *normalized vertical dispersion*, as calculated by Archery Analytics' data analysis software would be 4.0.

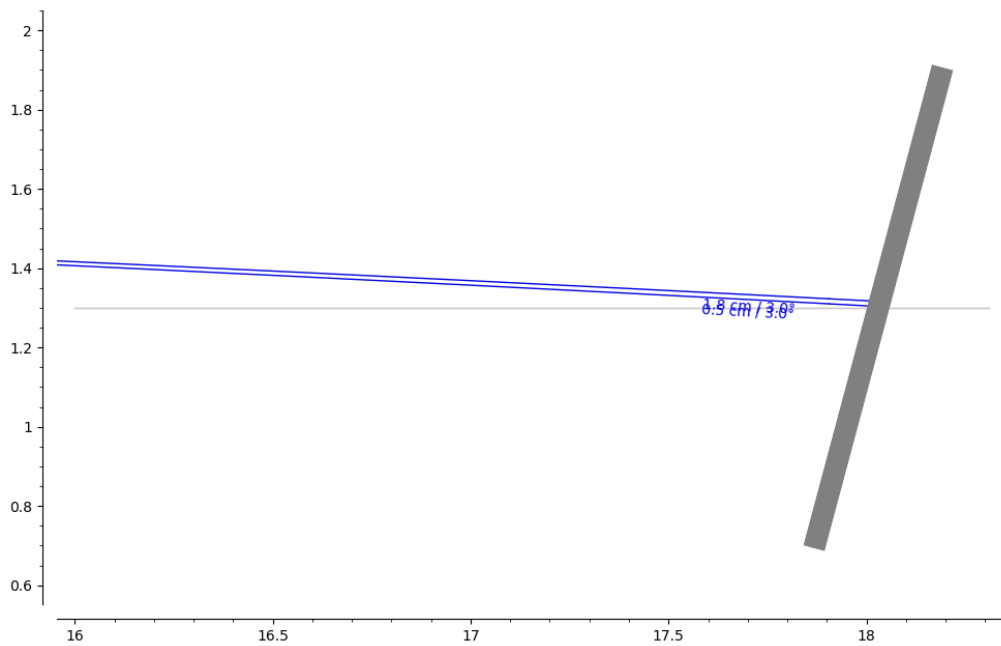


Figure 26: Target at 18 m

At 70 m distance, the normalized vertical dispersion is about 15 times as high as the respective number at 18 m. The difference in initial speed of 3 km/h makes a big effect on large distances.

This is the main reason why certain faults such as release faults can be identified by a difference in the normalized vertical dispersion. If, for example, you would alter the initial vertical angle of the arrow by 0.2° and keep the initial speed the same, the normalized vertical dispersion would be 5.0 in both 18 m and 70 m distance.

Now you may ask: is a speed difference of 3 km/h = 0.833 m/s a high value or not? With a typical recurve bow you would lose exactly this amount of initial speed when your fingers would release the deflection of the string about 1.4 cm before actually releasing it. And this can easily happen if your body tension is diminished or the mental focus is not to the point. On a 122 target at 70 m, this then makes a difference of 3 - 4 points.

c. Why do arrows need a spin?

Often, the fletches are put onto the shaft at a certain angle so that the air can attack the fletches and cause a spin of the whole arrow. However, if you think that a lot of spin is a big advantage, you probably are wrong.

Bullets, when fired, will receive a lot of spin. This is needed because the center of mass of bullets typically is behind the center point. Therefore, bullets would not have a stable behavior in the air without a large spin that stabilizes the bullet through a high angular momentum.

In contrast, arrows do have a center of mass that is actually before the shaft center, because we use a heavy head. This will lead to a stable flight, even without fletches. The aerodynamics are such that any deviation of the arrow from the tangent to the trajectory is immediately corrected for. The fletches just serve to provide a faster correction of the deviations, and this is the reason why in bare shaft tests fletched arrows behave slightly different compared to bare shafts.

So, why then do we want to have a spin of the arrows at all? The answer is that imperfections in releasing the arrows or the unavoidable oscillations of the arrow shafts will be balanced out while the arrow is slightly rotating on its flight. The arrows won't drift away, if they show e.g. a bit of porpoising. The time to arrive at a stable and smooth flight also becomes a bit shorter when the arrow is rotating slightly.

Now, if your arrows have got too much spin, another effect kicks in: Typically at the end of the trajectory the spin may become such strong, that the tail of the arrow is no longer closely following the trajectory due to the centrifugal force of the spinning arrow tail. This behavior is called 'tailspin'. Note that this effect is not caused by a slower speed of the arrow, but by a too high spin.

Tailspin will cause a higher normalized dispersion of the arrow groups on large distances, and therefore can be discovered when comparing this figure between short and long distances. The remedy is easy to get: just apply smaller angles to the fletches and reduce the spin.